Returns to Scale & Productivity

Evidence from the UK

Joel Kariel Anthony Savagar

Kent Workshop

Roadmap

Motivation

Theory

Data & Estimation

Model

What are Returns to Scale?

We define:

$$\gamma = \underbrace{\left(\frac{\partial C}{\partial y}\frac{y}{C}\right)^{-1}}_{\text{Inverse cost elasticity.}} = \frac{AC}{MC}.$$

Why do Returns to Scale matter?

1. Useful description of market structure, related to markups and profit shares.
2. Tightly linked to:
   - Firm growth and survival (Clementi and Palazzo 2016; Rossi-Hansberg and Wright 2007)
   - Long-run growth (Romer 1986)
   - Productivity (Gao and Kehrig 2020)

Returns to Scale and Productivity

Generally: $y = zF(k, \ )$, so returns to scale set exogenously:

$$y = \nu$$

Returns to Scale and Productivity

Generally: $y = zF(k, \;)$, so returns to scale set exogenously:

$$y = \nu$$

$$\uparrow \nu \;=\; \uparrow y \;=\; \uparrow z \;=\; \uparrow Z$$

Returns to Scale and Productivity

Generally: $y = zF(k, \ )$, so returns to scale set exogenously:

$$y = \nu$$

$$\uparrow \nu \quad = \quad \uparrow y \quad = \quad \uparrow z \quad = \quad \uparrow Z$$

Consider: $y = zF(k, \ ) - \varphi$, so returns to scale are *endogenous*:

$$y = \nu(1 + s_{\varphi})$$

Returns to Scale and Productivity

Generally: $y = zF(k, \ )$, so returns to scale set exogenously:

$$y = \nu$$

$$\uparrow \nu \quad = \quad \uparrow y \quad = \quad \uparrow z \quad = \quad \uparrow Z$$

Consider: $y = zF(k, \ ) - \varphi$, so returns to scale are *endogenous*:

$$y = \nu(1 + s_{\varphi})$$

Now, additional relationship:

$$\uparrow z \quad = \quad \uparrow y \quad = \quad \uparrow \gamma$$

Literature

- RTS theory:

  Feenstra (2003), Hall (1988), Kee (2002), Lashkari et al. (2019), and Ruzic and Ho (2019).

- RTS estimation:

  Ackerberg et al. (2015), Basu and Fernald (1996), Gao and Kehrig (2020), Hall (1990), Klette (1999), Levinsohn and Petrin (2003), and Linnemann (1999).

  - UK: $\gamma \leq 1$ in manufacturing up to 1990.

    Harris and Lau (1998), Haskel et al. (1995), and Oulton (1996).

What We Do

1. Estimate RTS across UK economy.
2. Show relationships between RTS, firm size, and productivity at firm-level.
3. Introduce firm dynamics model with endogenous RTS + imperfect comp.

Roadmap

Motivation

Theory

Data & Estimation

Model

Returns to Scale

1. Rearrange the profit function: $y = \mu(1 - s_\pi)$

Returns to Scale

1. Rearrange the profit function: $\gamma = \mu(1 - s_\pi)$
2. Log-linearise output: $\gamma = \varepsilon_y \; + \varepsilon_{yk}$

Returns to Scale

1. Rearrange the profit function: $\gamma = \mu(1 - s_\pi)$
2. Log-linearise output: $\gamma = \varepsilon_y + \varepsilon_{yk}$
3. Cost-minimisation: $\gamma = \nu(1 + s_\varphi)$

Cost Minimisation

Cost minimising rms solve:

$$C := \min_{K;L} \; wL + rK \quad \text{s.t.} \quad y \quad zF(K;L) \quad :$$

Combining FOCs and applying Euler's homogeneous function theorem

F( ) h.o.d. :

$$= \quad 1 + \frac{}{y} \quad :$$

Full derivation

Roadmap

Motivation

Theory

Data & Estimation

Model

Data & Estimation

Data: ARDx from ONS, 50,000 rms per year, 1998 - 2014.

Equation: $y_{it} = z_{it} + {}_{k}k_{it} + {}_{l}l_{it} + {}_{m}m_{it} + {}_{it}$

Method: control function approaches. More detail

Data & Estimation

Data: ARDx from ONS, 50,000 rms per year, 1998 - 2014.

Equation: $y_{it} = z_{it} + {}_{k} k_{it} + {}_{l} l_{it} + {}_{m} m_{it} + {}_{it}$

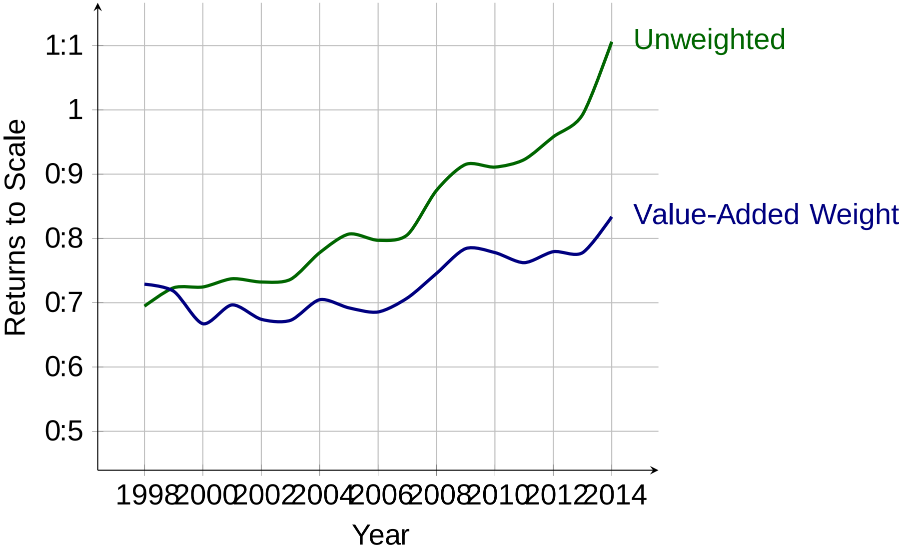
Method: control function approaches. More detail

But data includes revenue $p_{it}\, y_{it}$, not quantities y_{it} ...

Solution: $= \underbrace{(1 \quad s)}_{\{z\}}$ Markup Derivation

Revenue Elasticities

Returns to Scale in the UK



More Results

Wide distribution across industries (0.4 - 1.5) and rms.

Industry distribution   Firm distribution

Various methods produce similar results on aggregate, and over time

- Cobb-Douglas vs translog; investment vs materials proxy; gross-output vs value-added; di erent weights; remove outliers.

Returns to Scale and Productivity

$$= \quad 1 + \frac{\ }{y}$$

	Dependent variable: Returns to Scale				
Log TFP	0.003	0.007	0.915	0.944	0.635
	(0.026)	(0.026)	(0.094)	(0.093)	(0.053)
N	423,337	423,337	260,951	260,951	256,966
Year FE:		X		X	X
Firm FE:			X	X	X
Remove outliers:					X

Estimates statistically signi cant at levels of 0.1%: ***, 1%: **, 5%: *. Robust standard errors clustered at the level of the xed e ects included. Weighted by value-added at the rm level. Outliers are the top and bottom 1% of rms by returns to scale.

Roadmap

Motivation

Theory

Data & Estimation

Model

Why Do We Need A Model?

To reconcile:

- Rise in returns to scale (both via increasing markups and falling pro shares)
- Gradual rise in average rm size
- Productivity slowdown Figure
- Increase in share of unproductive rms

Dynamic Model (work-in-progress)

Standard rm dynamics model, with:

1. Output-denominated xed cost ! endogenous RTS.
2. Downwards-sloping demand curve ! (1) markups, (2) better account for rm size distribution.
3. Capital ! match estimation.

Dynamic Model (work-in-progress)

Standard rm dynamics model, with:

1. Output-denominated xed cost! endogenous RTS.
2. Downwards-sloping demand curve! (1) markups, (2) better account for rm size distribution.
3. Capital ! match estimation.

$$= py\ 1 \quad -$$

Both and are endogenous, depend on rm size, and rise as rms exi

Calibration

Model Results

- Declining business dynamism creates productivity slowdown, and returns to scale rise endogenously.
- Driven by rising markups.
- Firm distribution shifts left with long right-tail: lots of unproductive rms, a few superstars.

Results match UK experience:

RTS and TFP | Markups and Pro t Shares | Unproductive Firms | Falling Dynamism

Summing Up

1. Decreasing RTS in the UK, but has gradually risen
2. Negative relationship between RTS and productivity
3. Consistent withendogenous RTS arising from output-denominated xed cost
4. Simple homogeneous rm model not enough...
5. ...heterogeneous rm model with imperfect comp. matches UK facts

References I

Ackerberg, Daniel A. et al. (2015). Identi cation Properties of Recent Production Function Estimators . Econometrica 83.6, pp. 2411 2451.

Basu, Susanto and John Fernald (1996). Returns to Scale in U.S. Production Estimates and Implications . Social Science Research Network 1996, pp. 1 52.

Clementi, Gian Luca and Berardino Palazzo (2016). Entry, exit, rm dynamics, and aggregate uctuations . American Economic Journal: Macroeconomics 8.3, pp. 1 41.

De Ridder, Maarten (Mar. 2019). Market Power and Innovation in the Intangible Economy Discussion Papers 1907. Centre for Macroeconomics (CFM).

Edmond, Chris et al. (2015). Competition, markups, and the gains from international trade . American Economic Review 105.10, pp. 3183 3221.

References II

Feenstra, Robert C (2003). Advanced International Trade: Theory and Evidence. Princeton University Press.

Gao, Wei and Matthias Kehrig (Feb. 2020). Returns to Scale, Productivity and Competition: Empirical Evidence from US Manufacturing and Construction Establishments .

Hall, Robert (1988). The relation between price and marginal cost in US industry . Journal of political Economy 96.5, pp. 921 947.

(1990). Invariance Properties of Solow's Productivity Residual . Growth, Productivity, Unemployment: Essays to Celebrate Bob Solow's Birthday. MIT Press, pp. 71 112.

Harris, Richard and Eunice Lau (Apr. 1998). Verdoorn's law and increasing returns to scale in the UK regions, 1968 91: some new estimates based on the cointegration approach . Oxford Economic Papers 50.2, pp. 201 219.

References III

Haskel, Jonathan et al. (1995). PRICE, MARGINAL COST AND THE BUSINESS CYCLE. Oxford Bulletin of Economics and Statistics 57.1, pp. 25 39.

Kee, Hiau Looi (2002). Markup, Returns to Scale, and Productivity: A Case Study of Singapore's Manufacturing Sector. Tech. rep. 2857. Policy Research Working Paper, World Bank.

Klette, Tor Jakob (1999). Market Power, Scale Economies and Productivity: Estimates from a Panel of Establishment Data. The Journal of Industrial Economics 47.4, pp. 451 476.

Lashkari, Danial et al. (2019). Information Technology and Returns to Scale. 2019 Meeting Papers 1380. Society for Economic Dynamics.

References IV

Levinsohn, James and Amil Petrin (2003). Estimating production functions using inputs to control for unobservables. The Review of Economic Studies 70.2, pp. 317 341.

Linnemann, Ludger (Sept. 1999). Sectoral and aggregate estimates of the cyclical behavior of markups: Evidence from Germany. Review of World Economics (Weltwirtschaftliches Archiv) 135.3, pp. 480 500.

Olley, G. Steven and Ariel Pakes (1996). The dynamics of productivity in the telecommunications equipment industry. Econometrica 64.6, pp. 1263 1297.

Oulton, Nicholas (1996). Increasing Returns and Externalities in UK Manufacturing: Myth or Reality? The Journal of Industrial Economics 44.1, pp. 99 113.

Romer, Paul M. (1986). Increasing Returns and Long-Run Growth. Journal of Political Economy 94.5, pp. 1002 1037.

References V

Rossi-Hansberg, Esteban and Mark L. J. Wright (Dec. 2007). Establishment Size Dynamics in the Aggregate Economy. American Economic Review 97.5, pp. 1639 1666.

Rotemberg, Julio J. and Michael Woodford (1999). The cyclical behavior of prices and costs. Handbook of Macroeconomics. Ed. by J. B. Taylor and M. Woodford. Vol. 1. Handbook of Macroeconomics. Elsevier. Chap. 16, pp. 1051 1135.

Ruzic, Dimitrije and Sui-Jade Ho (Aug. 2019). Returns to Scale, Productivity Measurement, and Trends in U.S. Manufacturing Misallocation. INSEAD working paper.

Cost Minimisation

Cost minimising rms solve:

$$C := \min_{K;L} \quad wL + rK \quad \text{s.t.} \quad y \quad zF(K;L) \quad :$$

The solution yields:

$$C = \quad y \quad "_{yL} + "_{yK}$$

Applying Euler's homogeneous function theorem, we get:

$$C = z\,y \quad \frac{@y}{@L}\frac{L}{y} + \frac{@y}{@K}\frac{K}{y} \quad = z \quad \frac{@y}{@L}L + \frac{@y}{@K}K$$

$$= \quad (y + \quad)$$

It follows that the ratio of average to marginal costs is:

$$\frac{AC}{MC} = \frac{(1 + \quad s \quad)}{\qquad} = \quad (1 + \quad s \quad)$$

Return

Markup Derivation

The markup is the ratio of prices to the marginal cost:

$$= \frac{P}{MC}$$

If rms are cost-minimising, the measure of MC obtained by considering each input should be the same, so we can consider just one (e.g. the most exible: materials) (Rotemberg and Woodford 1999).

The marginal cost is the price of the input divided by its marginal product, and the result for Cobb-Douglas production (although it holds more generally):

$$MC = \frac{P_m}{\text{Marginal Product}_m} = \frac{P_m M}{{}_m Y}$$

And the markup can then be represented:

$$= P \frac{{}_m Y}{P_m M} = {}_m \frac{P Y}{P_m M} = {}_m ({}_m)^{1}$$

as the ratio of the exible input elasticity to the exible input share in revenue. Return

Control Function Approach I

Taking logarithms, we get:

$$y_{it} = {}_0 + {}_K k_{it} + {}_L l_{it} + {}_M m_{it} + {}_{it} :$$

where $\ln z_{it} = {}_0 + {}_{it}$.

Olley and Pakes (1996): timing of input choices; investment is a proxy fo

unobserved productivity shocks. Split up unobserved residual

${}_{it} = !_{it} + {}_{it}$, where $!_{it}$ is anticipated and ${}_{it}$ is an ex-post shock.

Control Function Approach II

Assumptions:

1. Information Sets: include current and past productivity shocks $\{\omega_{i\tau}\}_{\tau=0}^{t}$, but firms know nothing about future shocks.
2. First-Order Markov Shocks: productivity shocks follow a First-Order Markov Process, so $\omega_{it} = E(\omega_{it} | \omega_{i,t-1}) + \xi_{it}$.
3. Timing of Input Choices: previous period $i_{i,t-1}$ determines future capital k_{it}, whereas labour is chosen contemporaneously.
4. Scalar Unobservable: investment decisions $i_{it} = f_t(k_{it}, \omega_{it})$ have just one scalar unobservable ω_{it}.
5. Strict Monotonicity: investment decisions are strictly monotonic in the scalar unobservable ω_{it}, so $i_{it} = f_t(k_{it}, \omega_{it})$.

As i_{it} is strictly monotonic in unobserved anticipated shock, this function is inverted:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + f_t^{-1}(k_{it}, i_{it}) + \varepsilon_{it};$$

and the inverted function is approximated by a polynomial in k_{it}, i_{it}. Return

Returns to Scale Heterogeneity

Return

Returns to Scale in the UK

Return

Productivity Slowdown

Return

Calibration

Table: Calibration Parameters

	Parameter	Value	Target
Pre-set			
	Discount rate	0.9	Match annual IR
w	Wage	1	Normalise
	Capital elasticity	0.4	Production function estimation
	Labour elasticity	0.3	Production function estimation
	Depreciation rate	0.08	ONS
	AR(1) persistence	0.8	
	Elasticity of substitution	2	Edmond et al. (2015)
Calibrated			
z	AR(1) mean	3.11	Match log TFP
	Fixed cost	1.63	Match FC % in De Ridder (2019)
$_z$	AR(1) stdev	0.32	Match stdev of log TFP
c_e	Entry cost	200	Match exit rates
D	Exogenous demand	768.1	Match returns to scale

Return

Model Results I

Figure: Comparing model and data for RTS and TFP.

Return

Model Results II

Figure: Comparing model and data for markups and revenue elasticities.

Return

Model Results III

Figure: Unpro table Firms: area of productivity distribution (left) and share in model (right).

Return