Returns to Scale & Productivity in the Macroeconomy

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Disclaimers

Data

This work was produced using statistical data from ONS. The use of the ONS statistical data in this work does not imply the endorsement of the ONS in relation to the interpretation or analysis of the statistical data. This work uses research datasets which may not exactly reproduce National Statistics aggregates.

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Motivation

Motivation

- The UK has suffered low and stagnating productivity for several decades.
- Over this period returns to scale (RTS) have risen which typically increases productivity.
- We attempt to rationalise this counterintuitive observation.

This Paper

- Returns to scale in the UK economy have risen.
- Productivity in the UK has stagnated.
- Productivity and RTS are negatively related across industries.
- Neoclassical growth model with endogenous industry structure a la Hopenhayn is able to replicate this relationship.
- The mechanism relies on higher RTS reducing selection (making it easier for less productive firms to survive).

What are Returns to Scale?

- RTS capture the proportional change in output for a proportional change in inputs.
- It is the inverse cost elasticity which is the average cost (AC) to marginal cost (MC) ratio:

$$\text{RTS} = \frac{AC}{MC} \equiv \begin{cases} \text{Increasing returns,} & \text{if RTS} > 1 \\ \text{Constant returns,} & \text{if RTS} = 1 \\ \text{Decreasing returns,} & \text{if RTS} < 1 \end{cases}$$

- From $\pi = py - \mathcal{C}$ we have a common expression:

$$\text{RTS} = \mu \left(1 - \frac{\pi}{py}\right) \tag{1}$$

- μ is price over marginal cost markup and π/py is profit share.

Literature

Where do we fit?

- RTS estimation
  - U.S. – Hall (1988), Basu and Fernald (1996), Laitner and Stolyarov (2004), Ruzic and Ho (2019), and Gao and Kehrig (2021)
  - U.K. – RTS ≤ 1 in manufacturing up to 1990. Haskel, Martin, and Small (1995), Oulton (1996), and Harris and Lau (1998).
- Model: Restuccia and Rogerson (2008), Barseghyan and DiCecio (2011), Barseghyan and DiCecio (2016), and Gao and Kehrig (2021).

Model

Final Goods Producer

- Final goods producer solves

$$
\begin{aligned}
\Pi_t^F = \max_{y_t(\imath)} \quad & Y_t - \int_0^{N_t} p_t(\imath) y_t(\imath) d\imath \\
\text{s.t.} \quad & Y_t = N_t^{1+\epsilon} \left[\frac{1}{N_t} \int_0^{N_t} y_t(\imath)^{\frac{1}{\mu}} d\imath \right]^{\mu}
\end{aligned} \tag{2}
$$

- ϵ is love of variety or external scale economies.
- Optimality yields inverse-demand for firm:

$$
p_t(\imath) = N_t^{\frac{1+\epsilon-\mu}{\mu}} \left(\frac{y_t(\imath)}{Y_t} \right)^{\frac{1-\mu}{\mu}} . \tag{3}
$$

Intermediate Goods Producer

Timeline

1. A firm pays cost κ to enter.
2. It receives a productivity draw $\jmath \in [0, 1]$ from a Pareto distribution,
$$A(\jmath) = \frac{1}{(1-\jmath)^{\frac{1}{\vartheta}}}, \tag{4}$$
where $\vartheta > 1$ is the Pareto shape parameter.
3. Given productivity draw, it decides whether to be active or inactive.
   - Overhead cost ϕ causes some entrants to remain inactive.
4. All firms, both active and inactive, exit after one period.

Intermediate Goods Producer

Production

- Firm production function given by

$$y_t(\jmath) = A_t(\jmath)^{1-\nu} \left[k_t(\jmath)^{\alpha} \ell_t(\jmath)^{1-\alpha}\right]^{\nu}, \tag{5}$$

- Production labour is total labour less fixed overhead:

$$\ell_t(\jmath) = \ell_t^{\mathrm{tot}}(\jmath) - \phi. \tag{6}$$

- Intermediate goods producer solves

$$\pi_t(\jmath) = \max_{k_t(\jmath), \ell_t(\jmath)} p_t(\jmath) y_t(\jmath) - r_t k_t(\jmath) - w_t(\ell_t(\jmath) + \phi) \tag{7}$$

- Subject to production function and inverse demand.
- Firms charge constant markup μ of price over marginal cost:

$$\frac{p_t(\jmath)}{MC_t(\jmath)} = \mu. \tag{8}$$

Scaled Productivity and Profit

- From factor market equilibrium and inverse demand function.
- Ratio of firm size equals the scaled productivity ratio:

$$\frac{p_t(\jmath)y_t(\jmath)}{p_t(\imath)y_t(\imath)} = \frac{k_t(\jmath)}{k_t(\imath)} = \frac{\ell_t(\jmath)}{\ell_t(\imath)} = \frac{a_t(\jmath)}{a_t(\imath)},\ \forall \imath, \jmath, \tag{9}$$

$$\text{where } a_t(\jmath) \equiv A_t(\jmath)^{\frac{1-\nu}{\mu-\nu}}. \tag{10}$$

- Then profits are:

$$\pi_t(\jmath) = \left(1 - \frac{\nu}{\mu}\right) p_t(\jmath)y_t(\jmath) - w_t\phi \tag{11}$$

$$= w_t\phi\left(\frac{a_t(\jmath)}{a(J_t)} - 1\right) \tag{12}$$

- where J_t is productivity level at which profits are zero.

Free Entry

- Free entry equates the entry cost to expected profits:

$$\kappa = w_t \phi (1 - J_t) \left(\frac{\bar{a}(J_t)}{a(J_t)} - 1 \right) \tag{13}$$

- Under Pareto, average productivity is determined by J_t:

$$\frac{a(J_t)}{\bar{a}(J_t)} = 1 - \frac{1}{\vartheta} \left(\frac{1 - \nu}{\mu - \nu} \right) \tag{14}$$

- Free entry condition determines threshold productivity J_t.
- Once J_t is determined, average productivity follows:

$$\bar{A}(J_t) = \frac{\vartheta}{\vartheta - 1} (1 - J_t)^{-\frac{1}{\vartheta}}. \tag{15}$$

Returns to Scale

- From cost minimisation or log-linearising the production function: Cost min Log lin

$$\text{RTS}(\jmath) = \nu \left(1 + (1-\alpha) \frac{\phi}{\ell(\jmath)} \right). \tag{16}$$

- Effect of ν and ϕ on $\text{RTS}(\jmath)$ is ambiguous! Graph
- Average industry RTS is:

$$\overline{\text{RTS}} = \frac{\nu\Gamma + \mu\vartheta}{\Gamma + \vartheta}, \quad \text{where} \quad \Gamma = \frac{1-\nu}{\mu-\nu}. \tag{17}$$

- $\overline{\text{RTS}}$ increasing in ν, independent of ϕ.

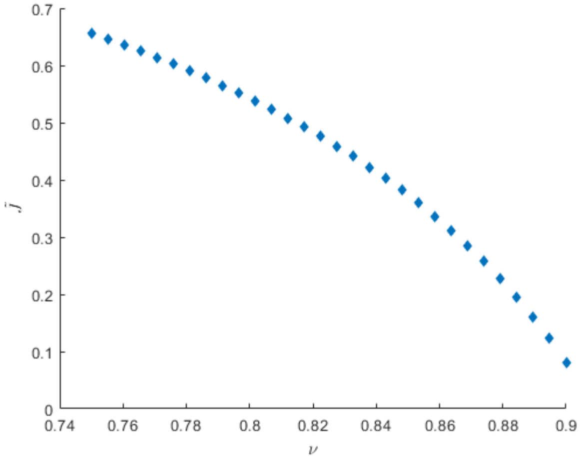
Model Analysis

Steady state analysis

- Under Pareto, model has an analytical steady state $\{\tilde{C}, \tilde{K}\}$.
- We conduct a comparative statics exercise.
- Our interest is the relationship between returns to scale and average productivity $\tilde{A}$.
- Ultimately we think about how two key parameters ν and ϕ affect these endogenous objects.

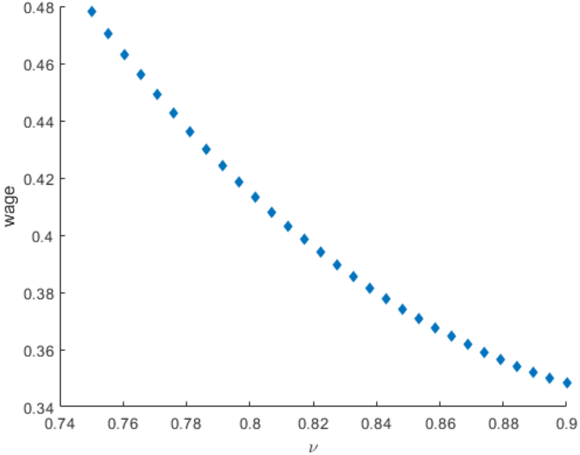
Steady State Analysis I

Figure 1: Threshold productivity $\tilde{J}$ and variable returns to scale ν.



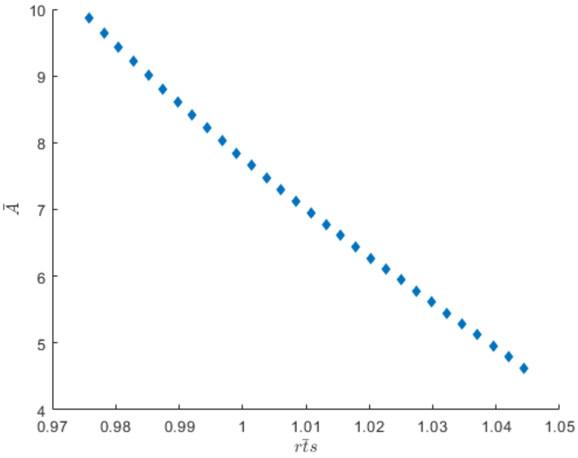
Steady State Analysis II

Figure 2: Wage and variable returns to scale ν.



Steady State Analysis III

Figure 3: Average productivity and average returns to scale.



Empirical Strategy

Data

- ARDx dataset is the UK's annual production survey.
- Runs from 1998 - 2014 and covers all sectors of the economy.
- 50,000 firms per year, 11m workers, 2/3 of GVA.
- Includes all large firms ($>$250 employees) and a representative sample of smaller firms.
- We use data on: gross revenue, value added, labour (# employees), materials and investment.
- We construct capital stock using the perpetual inventory method from firm-level investment data.

Estimation

- We estimate a Cobb-Douglas production function:

$$y_{it} = a_{it} + \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \epsilon_{it}$$

- The βs are elasticities to factor inputs

$$\text{RTS} = \beta_k + \beta_\ell + \beta_m \tag{18}$$

Mapping theory to data

- Endogeneity problem: cannot observe productivity a_{it}, which affects optimal input factor choices.
- Olley and Pakes (1996), Levinsohn and Petrin (2003), Wooldridge (2009), Ackerberg, Caves, and Frazer (2015), Grieco, Li, and Zhang (2016), and Gandhi, Navarro, and Rivers (2020). More detail

Empirical Results

Returns to Scale Levels

Comparison of Estimation Strategies

Table 1: Cobb-Douglas production function, 1998 - 2014

	Olley and Pakes (1996)	Levinsohn and Petrin (2003)	Ackerberg, Caves, and Frazer (2015)	Gandhi, Navarro, and Rivers (2020)
RTS	1.018	1.137	1.051	1.024
N	303,069	449,484	527,813	527,813

- Pooling all firms over all years.

Rising Sectoral Returns to Scale in the UK

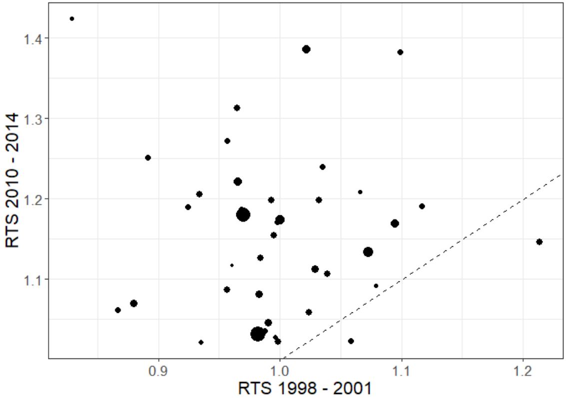


Figure 4: 2-Digit RTS Estimates 1998-2001 vs 2010-2014, GNR

- Point size represents the number of firms in that sector.
- Dotted line is 45 degree line: points above that line have higher RTS 2010-2014 than 1998-2001.
- Ackerberg, Caves, and Frazer (2015) Aggregate

Productivity

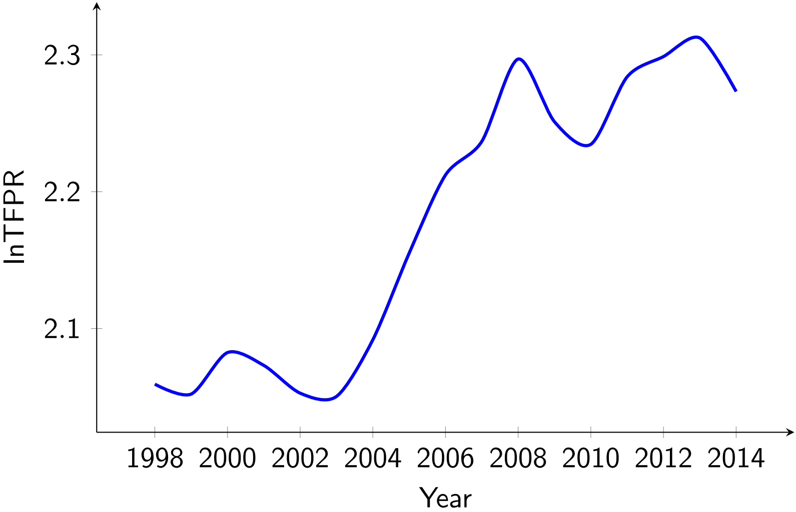


Figure 5: Aggregate TFPR

- Mean of firm productivity from GNR

Returns to Scale and Productivity

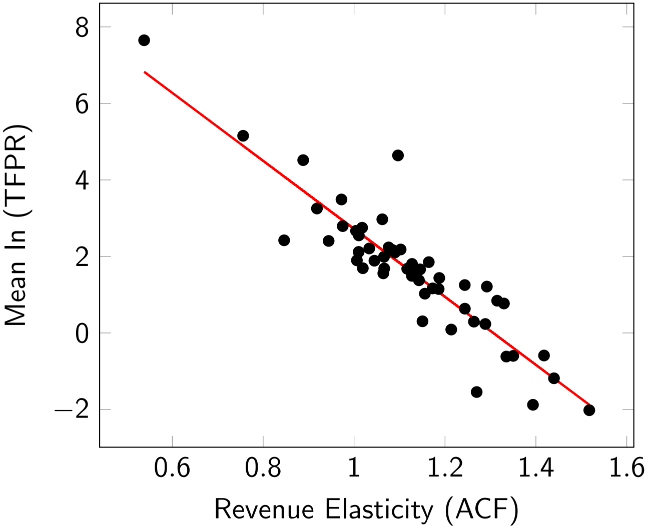


Figure 6: Relationship between RTS and TFPR

Summing Up

- In the UK productivity has stagnated whilst returns to scale have risen.
- We present a GE model that can produce a negative relationship between productivity and RTS.
- The model relies on a selection mechanism: higher RTS make it easier for low productivity 'laggards' to survive.

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Aggregation I

- Operating firms are a fraction of entering firms

$$N_t = E_t \int_{J_t}^{1} d\jmath = E_t(1 - J_t) \tag{19}$$

- The remaining aggregation follows as

$$K_t = E_t \int_{J_t}^{1} k_t(\jmath) d\jmath \tag{20}$$

$$L_t = E_t \int_{J_t}^{1} \ell_t(\jmath) + \phi d\jmath \tag{21}$$

- u_t is the fraction of aggregate labour that goes to production

$$u_t \equiv \frac{E_t \int_{J_t}^{1} \ell(\jmath) d\jmath}{L_t} \quad \text{and} \quad 1 - u_t = \frac{N_t \phi}{L_t}. \tag{22}$$

Aggregation II

- Aggregate output

$$Y_t = N_t^{1+\epsilon-\nu} \bar{a}(J_t)^{\mu-\nu} \left[K_t^{\alpha} \left(u_t L_t \right)^{1-\alpha} \right]^{\nu}. \tag{23}$$

- Factor market equilibrium

$$w_t = (1-\alpha) \frac{\nu}{\mu} \frac{Y_t}{u_t L_t} \tag{24}$$

$$r_t = \alpha \frac{\nu}{\mu} \frac{Y_t}{K_t} \tag{25}$$

- Zero profit condition

$$w_t = \left(1 - \frac{\nu}{\mu} \right) \frac{a(J_t)}{\bar{a}(J_t)} \frac{Y_t}{(1-u_t) L_t} \tag{26}$$

Return

Closing the model I

- The resource constraint

$$Y_t = C_t + I_t \tag{27}$$

- Entry fees are rebated to households by the government

$$T_t = E_t \kappa \tag{28}$$

- Profits and labour market clearing

$$\Pi_t = \Pi_t^F \tag{29}$$
$$L_t = L_t^s \tag{30}$$

Return

Control Function Approach I

Consider the Leontief production function:

$$Y_{it} = \min\{Z_{it} K_{it}^{\beta_k} L_{it}^{\beta_l}, M_{it}^{\beta_m}\} e^{\epsilon_{it}}.$$

where $Y_{it}, K_{it}, L_{it}, M_{it}$ represent revenue, capital stock, employment, and materials respectively, while the β's are the production elasticities. Z_{it} are ex-ante shocks. ϵ_{it} are ex-post shocks. Taking logarithms yields the "value-added" production function:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it}.$$

where $\ln z_{it} = \beta_0 + \omega_{it}$. Firms draw productivity ω_{it} which is unobserved by the econometrician, leading to potential omitted variable bias, as clearly the optimal firm input choices will be correlated with this variable.
We proceed with an explanation of the control function method of Ackerberg, Caves, and Frazer (2015), which makes assumptions on the timing of input choices to achieve identification, and uses materials expenditure as a proxy for unobserved productivity shocks. The following assumptions are required:

Control Function Approach II

1. **Information Sets:** firms' information sets I_{it} include current and past productivity shocks $\{\omega_{i\tau}\}_{\tau=0}^{t}$, but firms know nothing about future shocks. The ex-post shocks ϵ_{it} are expected to be zero on average: $\mathbb{E}\{\epsilon_{it}|I_{it}\} = 0$.
2. **First-Order Markov Shocks:** productivity shocks follow a First-Order Markov Process, so $\omega_{it} = \mathbb{E}(\omega_{it}|\omega_{i,t-1}) + \nu_{it}$, and $\mathbb{E}\{\nu_{it}|I_{it-1}\} = 0$.
3. **Timing of Input Choices:** firms accumulate capital according to $k_{it} = \kappa(k_{it-1}, i_{it-1})$ where investment i_{it-1} is chosen in period $t-1$. Labour l_{it} is chosen at period $t, t-1$ or in between. m_{it} is either chosen at the same time, or after l_{it} is chosen.
4. **Scalar Unobservable:** investment decisions $m_{it} = f_t(k_{it}, \omega_{it}, l_{it})$ have just one scalar unobservable ω_{it}, so there is no other across firm unobserved heterogeneity (e.g. adjustment costs, investment efficiency, input prices).
5. **Strict Monotonicity:** investment decisions are strictly monotonic in the scalar unobservable ω_{it}, so $m_{it} = f_t(k_{it}, \omega_{it}, l_{it})$.

Control Function Approach III

Given that investment is strictly monotonic in the unobserved anticipated shock, this function can be inverted, and then substituted into the production function:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(k_{it}, m_{it}, l_{it}) + \epsilon_{it} = \Phi_t(k_{it}, \omega_{it}, l_{it}) + \epsilon_{it}.$$

This inverted function is unknown, so is approximated by a polynomial. Running the first-step OLS regression yields an estimate of the composite term Φ_t. Clearly neither β_k nor β_l are identified in this stage, as both are contained in the composite term:

$$\widehat{\Phi}_t = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it}.$$

Production function parameters are estimated in the second stage. With the estimate of the composite term, estimates of the ex-ante productivity shock $\widehat{\omega}_{it}(\beta_k, \beta_l)$ can be computed for guesses of β_k, β_l. The implied $\widehat{\omega}_{it}(\beta_k, \beta_l)$ are non-parametrically regressed on their lag $\widehat{\omega}_{it-1}(\beta_k, \beta_l)$, and the residuals $\widehat{\nu}_{it}(\beta_k, \beta_l)$ are the implied innovations in productivity. Finally, the sample analogue of the moment condition $\mathbb{E}\{\nu_{it} k_{it}\} = 0$ is:

$$\frac{1}{N}\frac{1}{T}\sum_i \sum_t \widehat{\nu}_{it}(\beta_k, \beta_l) k_{it} = 0.$$

Control Function Approach IV

If labour is assumed to be chosen after $t-1$, l_{it} will generally be correlated with ν_{it}, so lagged labour is chosen as an additional moment condition. This procedure yields estimates $\widehat{\beta_k}, \widehat{\beta_l}$.

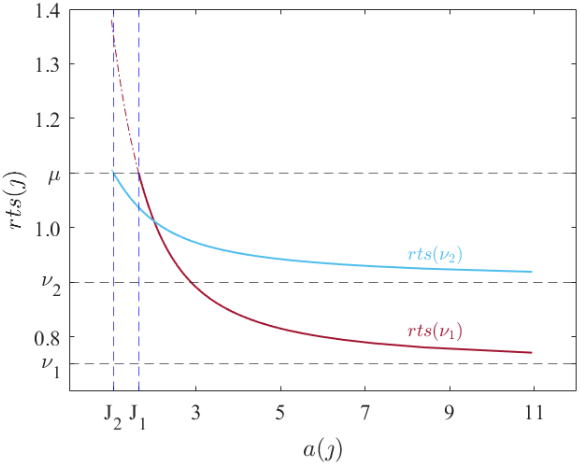
In order to obtain returns to scale at the level of the firm and year, a slightly different estimation procedure is required. I need to estimate time- and firm-specific output elasticities $\theta_{it}^k, \theta_{it}^l$. This can be achieved by generalising the production function to translog, as in De Loecker and Warzynski (2012):

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \beta_{ll} l_{it}^2 + \beta_{kk} k_{it}^2 + \beta_{lk} l_{it} k_{it} + \epsilon_{it}.$$

Then follow the same procedure as used with a Cobb-Douglas production function to estimate the coefficients and productivity process. The time- and firm-specific output elasticities are easily computed:

$$\widehat{\theta}_{it}^k = \widehat{\beta}_k + 2\widehat{\beta}_{kk} k_{it} + \widehat{\beta}_{lk} l_{it}.$$
$$\widehat{\theta}_{it}^l = \widehat{\beta}_l + 2\widehat{\beta}_{ll} l_{it} + \widehat{\beta}_{lk} k_{it}.$$

Returns to Scale and ν



Dotted blue lines represent productivity cut-offs. Dotted black lines show highest/lowest levels of returns to scale for operating firms. $\nu_1 = 0.75$ and $\nu_2 = 0.90$. Return

Aggregate Returns to Scale are Rising

Table 2: Cobb-Douglas production function, 1998 - 2014

	1998 - 2001	2002 - 2005	2006 - 2009	2010 - 2014
	Ackerberg, Caves, and Frazer (2015)			
RTS	0.988	1.081	1.046	1.061
N	153,874	144,465	108,619	120,855
	Gandhi, Navarro, and Rivers (2020)			
RTS	0.994	1.028	1.032	1.025
N	153,874	144,465	108,619	120,855

- Pooling all firms over all years and studying sub-periods.

Rising Sectoral Returns to Scale in the UK

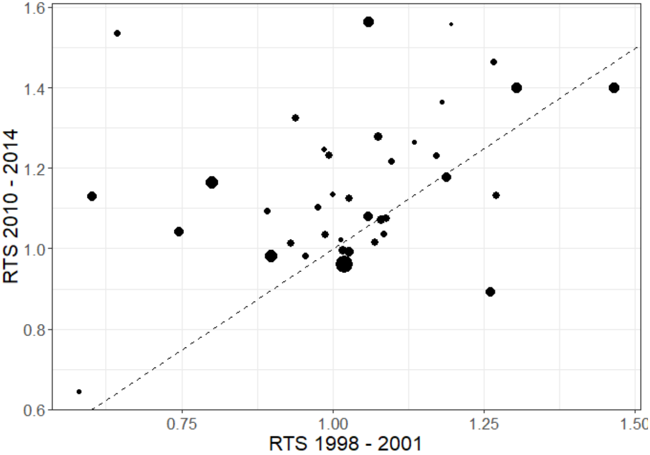
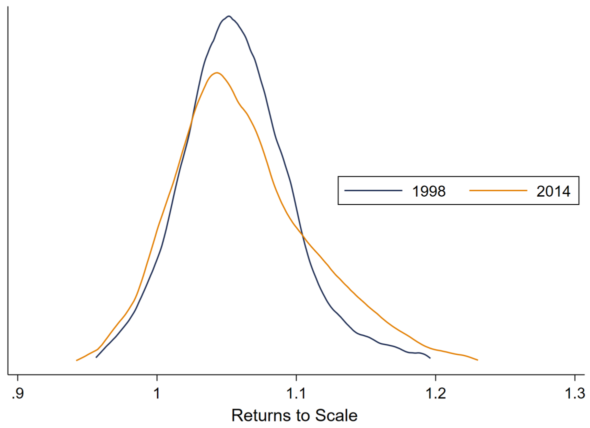


Figure 7: *Comparison of returns to scale using Ackerberg, Caves, and Frazer (2015) at 2-digit SIC level, from 1998 - 2001 to 2010 - 2014. Size of points represents the average number of firms in that sector in each period. Dotted line is 45 degree line: points above that line are consistent with a rise in returns to scale.* Return

Returns to Scale in the UK



Return

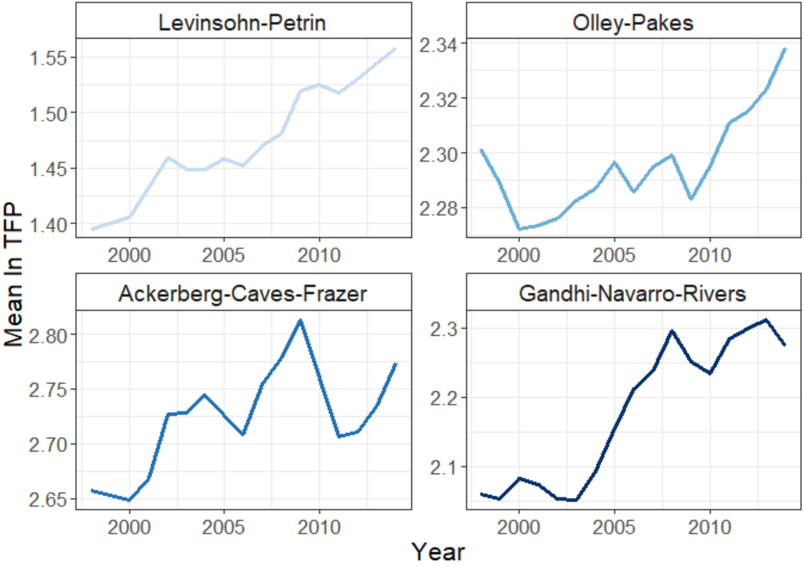
Sectoral Returns to Scale in the UK

Table 3: Cobb-Douglas production function, 1998 - 2014

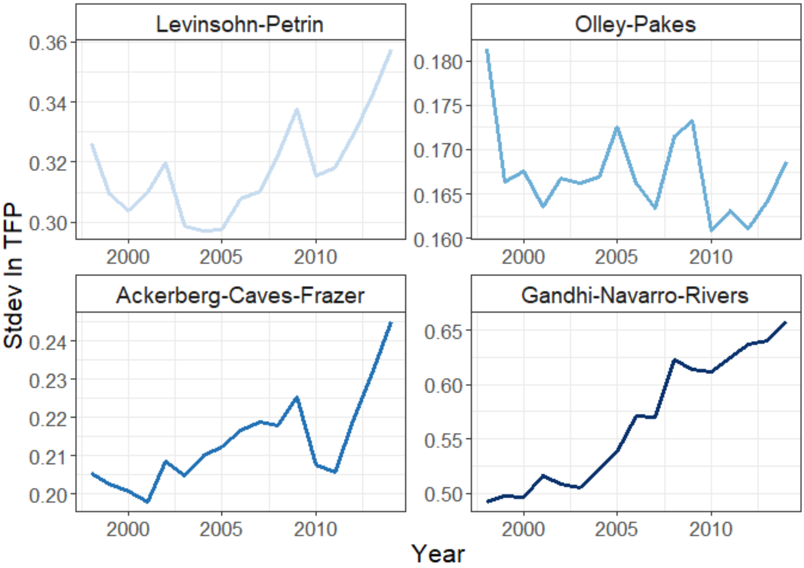
	Olley and Pakes (1996)	Levinsohn and Petrin (2003)	Ackerberg, Caves, and Frazer (2015)	Gandhi, Navarro, and Rivers (2020)
Manufacturing	1.252	1.121	**1.143**	**1.034**
Construction	1.025	0.805	**1.192**	**1.044**
Wh/Trade/Tran	1.027	1.009	**0.926**	**1.016**
Services	1.021	0.938	**1.067**	**1.015**

- RTS for Manufacturing and Construction is higher than for Wholesale/Trade/Transport and Services.

Aggregate Productivity Trends



Aggregate Productivity Dispersion



Returns to Scale and Productivity (GNR)

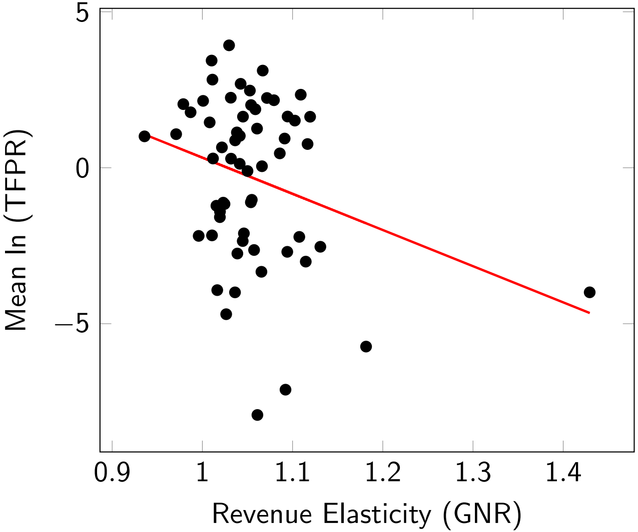


Figure 8: Relationship between Revenue Elasticity and TFPR

Returns to Scale Theory

$$\gamma = \frac{\mathcal{AC}}{\mathcal{MC}}.$$

We don't have data on costs! So how do we move forward?

1. Rearrange the profit function: $\gamma = \mu(1 - s_\pi)$
2. Cost-minimisation: $\gamma = \nu + \mu s_\phi$
3. Log-linearise output: $\gamma = \varepsilon_{y\ell} + \varepsilon_{yk}$
4. Model: $\gamma = \nu \left(1 + (1 - \alpha)\frac{\phi}{\ell}\right)$

Cost Minimisation

Cost minimising firms solve:

$$\mathcal{C} := \min_{\ell,k} \ w\ell^{\text{tot}} + rk \quad \text{s.t.} \quad y \geq zF(k, \ell^{\text{tot}} - \phi).$$

where $\ell = \ell^{\text{tot}} - \phi$ and $F(k, \ell)$ is h.o.d. ν. The familiar FOCs:

$$w = \lambda z F_k, \qquad r = \lambda z F_\ell$$

Applying Euler's homogeneous function theorem, we get:

$$\mathcal{C} = w\ell^{\text{tot}} + rk = \lambda z \left(F_\ell \left(\ell + \phi\right) + F_k k\right) = \lambda \nu y + \lambda z F_\ell \phi$$

It follows that the ratio of average to marginal costs is:

$$\frac{\mathcal{AC}}{\mathcal{MC}} = \frac{\lambda \nu y + \lambda z F_\ell \phi}{\lambda y} = \nu + \mu \underbrace{\frac{w\phi}{py}}_{s_\phi}$$

where s_ϕ is the fixed cost share in revenue. Return

Mapping theory to data I

- The linearization of model production function yields

$$\hat{y}_t = (1-\nu)\hat{A}_t + \alpha\nu\hat{k}_t + \nu(1-\alpha)\hat{\ell}_t. \tag{31}$$

- However we observe total labour ℓ_t^{tot} not production labour.
- Linearizing ℓ_t around steady-state, yields

$$\hat{\ell}_t = \left(1 + \frac{\phi}{\ell^*}\right)\hat{\ell}_t^{\text{tot}}. \tag{32}$$

- Therefore the equation we estimate is

$$\hat{y}_t = (1-\nu)\hat{A}_t + \alpha\nu\hat{k}_t + \nu(1-\alpha)\left(1 + \frac{\phi}{\ell^*}\right)\hat{\ell}_t^{\text{tot}} \tag{33}$$

- The sum of the coefficients on factor inputs corresponds to our model definition of returns to scale

$$\text{RTS}_t = \nu\left(1 + (1-\alpha)\frac{\phi}{\ell^*}\right). \tag{34}$$

Return (RTS) Return (Estimation)

Mapping theory to data I

Revenue Data

- In practice we have revenue data to measure output.
- Therefore the equation we estimate is

$$\widehat{p_t y_t} = (1-\nu)\hat{A}_t + \alpha\nu\hat{k}_t + \nu(1-\alpha)\left(1+\frac{\phi}{\ell^*}\right)\hat{\ell}_t^{\text{tot}} \qquad (35)$$

- The sum of the coefficients on factor inputs corresponds to our model definition of returns to scale

$$\text{RTS}_t = \nu\left(1+(1-\alpha)\frac{\phi}{\ell^*}\right). \qquad (36)$$

Return (RTS) Return (Estimation)